Early Journal Content on JSTOR, Free to Anyone in the World

This article is one of nearly 500,000 scholarly works digitized and made freely available to everyone in the world by JSTOR.

Known as the Early Journal Content, this set of works include research articles, news, letters, and other writings published in more than 200 of the oldest leading academic journals. The works date from the mid-seventeenth to the early twentieth centuries.

We encourage people to read and share the Early Journal Content openly and to tell others that this resource exists. People may post this content online or redistribute in any way for non-commercial purposes.

Read more about Early Journal Content at http://about.jstor.org/participate-jstor/individuals/early-journal-content.

JSTOR is a digital library of academic journals, books, and primary source objects. JSTOR helps people discover, use, and build upon a wide range of content through a powerful research and teaching platform, and preserves this content for future generations. JSTOR is part of ITHAKA, a not-for-profit organization that also includes Ithaka S+R and Portico. For more information about JSTOR, please contact support@jstor.org.

EMPIRICAL LAWS OF DEMAND AND SUPPLY AND THE FLEXIBILITY OF PRICES

SEVERAL months ago, on February 18, 1919, in New Orleans, the Southern States Cotton Acreage Reduction Convention held its meeting in which four hundred delegates from ten Southern states participated. The immediate object of the convention, as reported by the *New Orleans Times-Picayune*, was to give definiteness to a "movement to organize the cotton farmers, merchants and bankers of the entire South in reducing cotton acreage during the coming year, so as to free them from the shearing operations of the interests, American and foreign, which have been holding down the price of cotton." But the aspiration of many workers in the cause went beyond the planning for a single year and looked toward a permanent policy. Mr. Clarence Poe, the well-informed editor of the *Progressive Farmer* of Dallas, Texas, impressively urged, according to the *Times-Picayune*, that "the fight should be to secure permanently higher prices of cotton. . . . Our determination should be to put cotton on a level with other crops in the matter of wages paid for producing it. We should simply 'go on a strike' against raising cotton on the cheap-labor, child-labor starvation basis of production heretofore obtaining." The chairman of the convention, Governor R. G. Pleasant of Louisiana, expressed a common sentiment in saying "we must organize in every ward in the South and show the people that a reduction of at least one-third of the cotton acreage is necessary, proper and profitable. . . . Planting more than two-thirds of our present acreage in cotton is waste, and waste is criminal. The extra one-third should be devoted to feedstuffs, cattle, swine and sheep." Mr. J. M. Parker, "the federal food administrator for Louisiana, brought to the convention one thousand copies of a pamphlet setting forth to the cotton farmers, merchants and bankers of the South his plan of meeting the situation." According to

Mr. Parker "an intimate knowledge of many years of conditions, both as a cotton producer and as a large handler of cotton" justified his statement "which, he says, can be readily proven" that "by selling 70 per cent. of their crop the farmers would really receive more money than they would by selling 100 per cent. . . ."

The impassioned editor from Texas, Mr. Poe, to whom reference has already been made, concurred in the recommendation to reduce the acreage one-third in 1919 and supported his plea for a permanent policy of small acreage by describing the state of cotton farming from 1890–1900: "What happened then was that we managed to keep soul and body together by taking the price paid for cotton and mortgaging the farms we owned to make up the difference between the price of cotton and the cost of living. I know, because my father and I were making cotton then, as I am now, and the old farm which had been in the family for three generations had to be mortgaged to meet living expenses. The period from 1890 to 1900 was a period of wholesale heartsickening change from land ownership to tenancy in the South, and this is the explanation: We were simply selling cotton for less than cost and paying for the privilege by giving away our homes to boot." But this is not the darkest view of the situation. Mr. Poe tells us further that cotton "has brought a low price because it has been largely a child-labor crop; largely also a woman-labor crop; and because the labor which has produced it has lived largely under slum conditions. . . . While children in all other states and countries have been going to school and getting an education, together with all the advantages and benefits which education confers, we have been keeping our children in the fields and letting them grow up in ignorance—comparative ignorance at the very least. . . . But the main point is this: That we have done all this for nothing. We have made this shameful sacrifice of the immortal minds of our boys and girls for nothing. Why do I say for nothing? Because by keeping our children in the fields instead of in the schools, we have simply produced larger crops than the world was willing to pay high prices for. And the net result is that we here in the South have gotten no more for making big crops of cotton and keeping our children in ignorance to do it than we should have gotten for the smaller crops of cotton we should have made if we had sent our children to school as all the rest of the world is doing."

The course of the Southern planters is giving concern to manufacturers everywhere. For example, we read in the *New York Times* of June 9, 1919, "British textile manufacturers are chafing under the necessity of buying so largely from America, and the threat of the cotton planters to reduce their acreage has aroused their resentment. . . . . Since the signing of the armistice many manufacturers from the Lancashire district have made visits to India, Japan, and South America," and the "manufacturers . . . have repeatedly expressed the view that the world's markets are bare of cloth."

Here certainly is a staggering conjuncture of facts: "The world's markets bare of cloth", and the producers of the raw material organizing to curtail their output in order that, immediately, they may increase their profits and, ultimately, raise the standard of living of their women and children. One of these facts is *fait accompli*: "The world's markets are bare of cloth." But as yet the plans for a systematic reduction of the acreage have not been successfully put into operation, and there is great need to consider the basic arguments of the planters as well as the conditions in which their discontent has its origin.

Is it true that, by reducing their acreage, the farmers are likely to increase the amount of their profits? Is it possible to calculate the approximate production that would yield the farmers the maximum return? To answer these questions one must be able to forcast the variations of price that will follow upon variations in the output; to estimate the marginal price that is necessary to call forth any stated supply; and to compute the aggregate profits corresponding to different amounts of product. These desiderata are not peculiar to the problem before the Southern planters,[1] but they occur again

---

[1] An illustration may be given from President Wilson's address to Congress on August 8, 1919, embodying recommendations designed to reduce the cost of living:

TABLE I

DATA FOR THE EMPIRICAL LAW OF DEMAND FOR COTTON

(One Year Base)

Year	Equivalent 500 Pound Bales, Gross Weight (Millions of Bales)	Production-Ratio	Percentage Change in Production	Price per Pound Upland Cotton (Cents)	Price-Ratio	Percentage Change in Price
1889	7.47		. .	11.5	.	. .
1890	8.56	115	+15	8.6	75	—25
1891	8.94	104	+ 4	7.3	85	—15
1892	6.66	75	—25	8.4	115	+15
1893	7.43	112	+12	7.5	89	—11
1894	10.03	135	+35	5.9	79	—21
1895	7.15	71	—29	8.2	139	+39
1896	8.52	119	+19	7.3	89	—11
1897	10.99	129	+29	5.6	77	—23
1898	11.44	104	+ 4	4.9	88	—12
1899	9.35	82	—18	7.6	155	+55
1900	10.12	108	+ 8	9.3	122	+22
1901	9.51	94	— 6	8.1	87	—13
1902	10.63	112	+12	8.2	101	+ 1
1903	9.85	93	— 7	12.2	149	+49
1904	13.44	136	+36	8.7	71	—29
1905	10.58	79	—21	10 9	125	+25
1906	13.27	125	+25	10.0	92	— 8
1907	11.11	84	—16	11.5	115	+15
1908	13.24	119	+19	9.2	80	—20
1909	10.00	76	—24	14 3	155	+55
1910	11.61	116	+16	14.7	103	+ 3
1911	15.69	135	+35	9.7	66	—34
1912	13.70	87	—13	12.0	124	+24
1913	14.16	103	+ 3	13.1	109	+ 9

and again in practical as well as in theoretical economic questions, and for that reason they should receive a general abstract treatment. The primary purpose of the following pages is to offer suggestions in regard to the more general problems, but it is hoped that light may thereby be thrown upon the perplex-

---

"In a few foodstuffs the prices had declined, but in nothing like the proportion in which supply had increased.

"For example, the stock of canned tomatoes had increased 102 per cent., and yet the price had declined only 25 cents per dozen cans. In some cases there had been the usual result of an increase of price following a decrease of supply, but in almost every instance the increase of price had been disproportionate to the decrease in stock." What would be the normal increase in price corresponding to the indicated reduction of stocks?

ing and momentous question which is being agitated among the cotton planters.

Preliminary Notions

In the accompanying Table I are figures relating to the production and the price of cotton in the United States from 1889

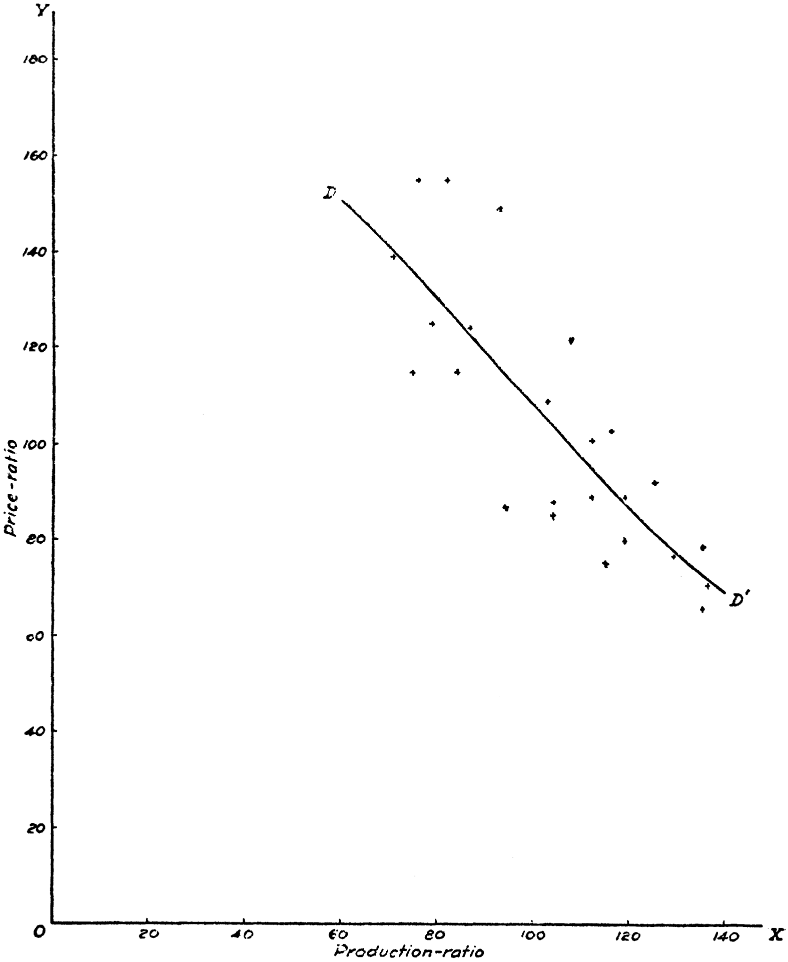

Figure 1. The law of demand for cotton.
$Y = 173.6 + .377X - .01603X^2 + .0000572X^3.$

to 1913. The third column of the Table expresses the ratio of the production of any one year to the production of the preceding year, and the sixth column expresses the ratio of the mean price of any one year to the mean price of the preceding year. If these two columns of ratios are plotted as in Figure 1, where the production-ratios are measured on the horizontal line and the price-ratios on the perpendicular line, the series of scattered points depicted in the Figure will be obtained. The smooth curve passing among the points is the graph of the mathematical equation,

$$Y = 173.6 + .377X - .01603X^2 + .0000572X^3,$$

which gives a general description of the relation of the price-ratios to the production-ratios. We learn from the smooth curve, which may be called the graph of the law of demand, the probable change in the price of cotton that, before 1914, would have followed upon a change in production. For example, the ratio of the production in 1914 to that of 1913 was 114, and consequently, in normal times, the probable ratio of the price in 1914 to that of 1913 would have been expressed by 93.

The data of Table I might have been treated in a different way. Column 4 expresses the percentage change in the production of any one year over the production of the preceding year, and column 7 gives similar information regarding prices. The mathematical equation describing the relation between these two series of figures is

$$y = 8.2 - 1.113x + .00113x^2 + .0000572x^3,$$

where x expresses the percentage change in the amount of production, and y expresses the percentage change in the corresponding price. This device for dealing with the data may be called the *method of percentage changes*. It is to be distinguished from the contrivance described a moment ago to which we shall refer as the *method of progressive ratios*.

A very valuable characteristic of these two methods is that we may pass, by a simple algebraic substitution, from the results obtained by the method of percentage changes to the results derived by the method of progressive ratios and *vice versa.* The equation to the law of demand when the method of progressive ratios is followed is

$$Y = 173.6 + .377X - .01603X^2 + .0000572X^3,$$

and when the method of percentage changes is used, the equation is

$$y = 8.2 - 1.113x + .00113x^2 + .0000572x^3.$$

If in the latter we put $y = Y - 100$, $x = X - 100$, we obtain the former equation.

From the point of view of regulating the amount of agricultural output with the object of bettering the farmer's condition, a matter of the first importance is the information given by the slope of the curve of demand. In Figure 2 are two hypothetical curves in one of which—the continuous curve—the slope is great, while in the other—the dotted curve—the slope is small. From the run of the two curves it is clear that for a given restriction of output, the price will rise sharply in the continuous curve and only moderately in the dotted curve. If,

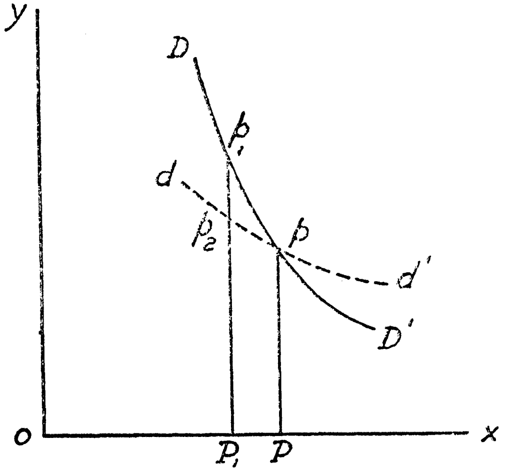

Figure 2.

for example, the production should be reduced from $O\,P$ to $O\,P_1$, the price, in the case of the smooth curve, would rise from $P\,p$ to $P_1\,p_1$; while, in the case of the dotted curve, it would rise only to $P_1\,p_2$. If the equation to the law of demand is given in either of the two forms which we have already described, the slope of the curve of demand at any point may be determined. Since the equation to the law of demand for cotton, in the percentage form, is,

$$y = 8.2 - 1.113x + .00113x^2 + .0000572x^3$$

the slope of the curve is $\frac{dy}{dx} = -1.113 + .00226x + .0001716x^2$, which may be computed for any given value of x, e. g., when $x = 10$, $\frac{dy}{dx} = -1.073$. The same result for the slope will be obtained if we take the equation in the form given by the method of progressive ratios. The appropriate equation for cotton, we have ascertained, is

$$Y = 173.6 + .377X - .01603X^2 + .0000572X^3,$$

and therefore

$\frac{dY}{dX} = .377 - .03206X + .0001716X^2$, which, when $X = 110$, gives $\frac{dY}{dX} = -1.073$.

The technical term *flexibility of prices* may be used to describe the variation of prices following upon a variation in the quantity of production, and we shall understand by this term the degree of bending in prices consequent upon an infinitesimal variation in the quantity of the output. We shall measure the degree of flexibility of prices by the *coefficient of flexibility*, which we shall define as the ratio of the relative change in price to the relative change in the supply of the commodity when both changes are infinitesimal. Symbolically, if the equation to the law of demand in the percentage-change form is $y = f(x)$, then the coefficient of flexibility of prices is $\frac{dy}{dx} = f'(x)$; and if the law of demand as given by the method of progressive ratios is $Y = F(X)$, the coefficient of flexibility of prices is $\frac{dY}{dX} = \mathrm{F}'(X)$. We have already seen that these values $\frac{dy}{dx}, \frac{dY}{dX}$ are represented graphically by the respective slopes of the two types of curves. The letter f may be used to represent the coefficient of flexibility of prices, and we shall agree to speak of prices as being flexible when f is numerically greater than unity; and as being inflexible, when f is numerically less than unity. A while ago, when we were discussing the law of demand for cotton in the percentage-change form, we found that $\frac{dy}{dx} = -1.113 + .00226x + .0001716x^2$. If, in this equation, we put $x = 0$, we have $\frac{dy}{dx} = f = -1.113$, and since this value is numerically greater than unity, we say that the prices of cotton are flexible.

Whether the prices of a particular commodity are flexible or inflexible is of critical importance in the problem of determining the quantity of output which will yield the producer the maximum return in profits. For, if the prices should be inflexible, we should know at once that, assuming there is no secular trend in prices, an increase of one per cent. in the quantity of commodity produced would be followed by a fall of less than one per cent. in the price, and, consequently, as far as concerned the demand for the commodity—for the moment ignoring the question of the cost of production—it would be to the advantage of the producer to increase his output. If, on the other hand, prices are flexible, a reduction in the supply of one per cent. would be followed by more than a one per cent. rise in price, and the farmer would find his profit in curtailing the amount of his production.

If in any particular case, let us say the production of cotton, it should be found that prices are flexible and the inference be drawn that the farmer should reduce his acreage, the question would immediately arise as to the amount by which his output should be reduced. The farmer's aim will obviously be so to restrict his production that, with the resulting increase of price, he will have a surplus over the costs which will be the maximum surplus obtainable under the existing law of demand and the prevailing costs of production. Let us for a moment ignore the law of supply and consider whether it is possible to ascertain the amount of production which, according to the law of demand, will yield the maximum income. If we take the law of demand in the ratio-form described in Figure 1, we have a means of solving the problem. In the equation to the law of demand in this form,

$$Y = 173.6 + .377X - .01603X^2 + .0000572X^3,$$

where X describes the ratio of the production of a given year to the production of the previous year, and Y gives the corresponding ratio of the price of the given year to the price of the preceding year. The goal of the farmer will be so to determine X that the product $X\ Y$ shall be a maximum. According to the mathematical rule for finding the maximum, the above product $X\ Y$ will have its greatest value when $\frac{d\,(X\ Y)}{d\,(X)} = 0$; that is, when

$$173.6 + .754X - .04809X + .0002288X^3 = 0.$$

The solution of this last equation shows that $X\ Y$ will be a maximum when $X = 98$.

Here, then, we reach an arresting conclusion. If cotton were a free gift of nature, costing nothing whatever for its production, it would be to the advantage of the owners, according to the preceding reasoning, to reduce the quantity placed upon the market by two per cent. But does the reasoning we have traversed justify so fateful a conclusion? In the presentation of our preliminary notions we have had occasion to speak of (1) the coefficient of flexibility of prices; (2) the empirical law of demand; (3) the empirical law of supply; (4) the economic amount of production, or the amount of output that will yield the planters the maximum income. In order to gain a more comprehensive view of the problem we shall consider these questions more in detail.

The Coefficient of Flexibility of Prices

We have defined the coefficient of flexibility of prices as the ratio of the relative change in price to the relative change in the supply of the commodity when both changes are infinitesimal. Symbolically, if x represents the percentage change in the amount supplied, y the corresponding percentage change in price, and f the coefficient of flexibility of prices, then $f = \frac{d\,y}{d\,x}$, and the value of f will depend upon (1) the method of measuring the raw values of percentage change in commodity and price, and (2) the type of equation chosen to represent the connection between x and y.[1] In the construction of Table I, our raw data were the percentage change in production of any one year over the production of the preceding year and the corresponding percentage change of price of the given year over the price of the previous year. But we might have taken the percentage change of production of any one year over the average production of the two preceding years, or of the three, or four, or more preceding years,—together with the corresponding percentage changes of price. We might have used as the foundation of our computations an arrangement of raw data illustrated in Table II, where the percentage changes in production and in price are calculated with reference to a four-years base.

---

[1] Perhaps some justification should be offered for the introduction of the new term *flexibility of prices*. It might be said that the coefficient of flexibility of prices is simply the reciprocal of the coefficient of elasticity of demand. I should gladly agree to the criticism and should point out that in 1914, in *Economic Cycles*, chap. iv, and in 1917, in *Forecasting the Yield and the Price of Cotton*, chap. v, I had done my best to give to the ideas of elasticity of demand and of law of demand, as we inherited them from the masters of statical economics, a concrete, dynamic significance. But there will probably always be co-workers who will prefer to restrict the term elasticity of demand to a statical meaning, and this is one reason for the use of the term flexibility of prices. A second reason is that, just as in trigonometry a facilitation of thought has been gained by having for the sine, tangent, and secant, separate names for their reciprocals,—cosecant, cotangent, and cosine—so in many economic problems real assistance will be afforded by keeping the mind upon the aspect of phenomena represented by flexibility of prices rather than upon that represented by elasticity of demand. The former term sees the movement of prices primarily from the point of view of the producer; the latter, primarily from the point of view of the consumer.

Our first query, therefore, in case of an alleged value of f, refers to the breadth of base of the raw data upon which the computation rests, whether the percentage changes have reference to a single year, or to a mean of several years. If it should be said that the price of cotton is flexible and the acreage should be reduced, we should wish first of all to know whether the computed flexibility was based upon a simple year to year change or upon a change covering a term of years. For one purpose it may be desirable to compute f in a particular way, and, for another purpose, in quite a different way.

Empirical Laws of Demand

Whether the raw data are taken with a base of one or more years, we should in each case have to choose a type of curve to use in connecting the variables x and y. The simplest assumption would be that the relation between x and y is linear and may be represented by $y = a\,x + b$, where the constants a and b are then derived from the raw data. But while this type of curve is simple, it may not fit the observations, and any conclusion which might be derived from the assumed law of demand would, where the fit is poor, probably be either inadequate or erroneous. We have no *a priori* knowledge as to the appropriate type of curve, but we do know that whatever may be the proper type it is representable by means of the series[1] $y = a + b\,x + c\,x^2 + d\,x^3 + \ldots$, and that the greater the number of terms in the series, the better the curve will fit the observations. Our object should be to retain a simple working equation by taking as few terms in the series as will give a fit sufficiently good for the problem in hand.

In the subsequent part of this paper the typical equation is

---

[1] This follows from Maclaurin's theorem that $f(x) = f(o) + x\,f'(o) + \frac{x^2}{\underline{|2}} f''(o) + \frac{x^3}{\underline{|3}} f'''(o) + \ldots$

TABLE II

DATA FOR THE EMPIRICAL LAW OF DEMAND FOR COTTON

(Four Years Base)

Year	Equivalent 500 Pound Bales, Gross Weight (Millions of Bales) P	Production-Ratio $P/\bar{P}_4$	Percentage Change in Production $\frac{P-\bar{P}_4}{\bar{P}_4}$	Price per Pound Upland Cotton (Cents) p	Price-Ratio $p/\bar{p}_4$	Percentage Change in Price $\frac{p-\bar{p}_4}{\bar{p}_4}$
1886 . .	6.31	. .	. .	10.3	. .	. .
1887 . .	6.88	. .	. .	10.3	. .	. .
1888 .	6.92	. .	. .	10.7	. .	. .
1889 . .	7.47	. .	. .	11.5	. .	. .
1890 . .	8.56	124	+24	8.6	80	—20
1891 . .	8.94	120	+20	7.3	71	—29
1892 . .	6.66	84	—16	8.4	88	—12
1893 . .	7.43	94	— 6	7.5	84	—16
1894 . .	10.03	127	+27	5.9	75	—25
1895 . .	7.15	87	—13	8.2	112	+12
1896 . .	8.52	109	+ 9	7.3	97	— 3
1897 . .	10.99	133	+33	5.6	78	—22
1898 . .	11.44	125	+25	4.9	73	—27
1899 . .	9.35	98	— 2	7.6	117	+17
1900 . .	10.12	101	+ 1	9.3	148	+48
1901 . .	9.51	91	— 9	8.1	119	+19
1902 .	10.63	105	+ 5	8.2	109	+ 9
1903 . .	9.85	99	— 1	12.2	147	+47
1904 . .	13.44	134	+34	8.7	93	— 7
1905 . .	10.58	97	— 3	10.9	117	+17
1906 . .	13.27	119	+19	10.0	100	0
1907 . .	11.11	94	— 6	11.5	111	+11
1908 . .	13.24	109	+ 9	9.2	89	—11
1909 . .	10.00	83	—17	14.3	137	+37
1910 . .	11.61	98	— 2	14.7	131	+31
1911 . .	15.69	137	+37	9.7	78	—22
1912 . .	13.70	108	+ 8	12.0	100	0
1913 . .	14.16	111	+11	13.1	103	+ 3

assumed to be $y = a + b\,x + c\,x^2 + d\,x^3$, and when we express the equation in the percentage-change form we use the small letters x and y, and when we express it in the form of progressive ratios we use the capital letters X and Y. Now it makes no difference whether the percentage changes in the raw data have, as their base, quotations for one year or for an average of several years, we may always pass from the law of demand in the percentage form to the law of demand in the form of progressive ratios and *vice versa*. For example, the law of demand for cotton derived from percentage changes with a base of four years is

$$y = 17.1 - 1.099x - .06566x^2 + .0019275x^3.$$

Put $y = Y - 100$, $x = X - 100$, then

$$Y = -2357.1 + 69.858X - .64391X^2 + .0019275X_3,$$

which is the law of demand in the form of progressive ratios. The method of arranging the raw data for computing these equations is given in Table II.

In concluding the preceding section we found that it is wise always to ask whether an alleged value of the coefficient of flexibility of prices is derived from raw data with a base of one year or of a mean of several years. With reference to cotton we may now see that the value of f is practically the same whether the law of demand is computed from raw material with a base of one year or a base of a mean of several years. In the former case the law of demand is

$$y = 8.2 - 1.113x + .00113x^2 + .0000572x^3,$$

which gives $f = \frac{dy}{dx} = -1.113 + .00226x + .0001716x^2$,

and when $x = 0$, $f = -1.11$.

The law of demand with a four-years base is

$$y = 17.1 - 1.099x - .06566x^2 + .0019275x^3,$$

which gives $f = \frac{dy}{dx} = -1.099x - .13132x + .0057825x^2$,

and when $x = 0$, $f = -1.10$.

The practical significance of this constancy in the value of f, which is numerically greater than unity, is to fortify the conclusion that the cotton farmers will find their interest in decreasing the amount of their production.

Empirical Laws of Supply

In treating this question a distinction must be made between (1) the empirical law of supply under circumstances of free competition, and (2) the empirical law of supply under circumstances of a deliberate restriction of the amount of production.

We have seen that the present movement among the cotton farmers takes the form of an agreement to reduce their acreage as a means of limiting the amount of their production. The aggregate production, it is true, is dependent upon the yield per acre as well as upon the acreage, but the farmers have no way of estimating, before the planting, the probable yield per acre, and their effort at regulating supply is limited to increasing or decreasing the acreage. Our problem therefore resolves itself into a search for the empirical law according to which the farmers regulate the area planted in cotton.

In Table III material is given with which to treat the problem. We would suppose that one important factor leading the farmers to increase or to decrease the number of acres planted is the movement of prices in the preceding years. If the price of cotton has been falling, fewer acres will be seeded in that crop; and, on the other hand, if prices have been rising, there is likely to be an increase in the acreage. In Table III data are given with which to compute the relation between the two variables, namely, the percentage change in the acreage of a given year over the acreage of the preceding year, and the percentage change in the price of cotton from the price prevailing two years before the current year to the price the year before the current year. The correlation between the two variables is $r = .532$, and the equation expressing their relation is

$$y = .375x + 2.76.$$

This equation gives the empirical law of the variation of cotton acreage under free competition. For any given percentage change in the price of the preceding year x, we may compute, by means of the equation, the probable percentage change in the acreage y. We may also observe that, just as we could pass from the results of the law of demand in the percentage-

TABLE III

DATA RELATING TO THE EMPIRICAL LAW OF SUPPLY OF COTTON

(Conditions of Free Competition)

Year	Acreage of Cotton (Thousands of Acres)	Acreage-Ratio	Percentage Change in Acreage	Price per Pound Upland Cotton (Cents)	Price-Ratio	Percentage Change in Price
1888	. . .	. .	. .	10.7	. .	. .
1889	20,180	. .	. .	11.5	107	+ 7
1890	21,886	108	+ 8	8.6	75	—25
1891	23,876	109	+ 9	7.3	85	—15
1892	15,228	64	—36	8.4	115	+15
1893	23,837	157	+57	7.5	89	—11
1894	24,959	105	+ 5	5.9	79	—21
1895	21,896	88	—12	8.2	139	+39
1896	32,823	150	+50	7.3	89	—11
1897	28,861	88	—12	5.6	77	—23
1898	25,174	87	—13	4.9	88	—12
1899	24,278	96	— 4	7.6	155	+55
1900	24,982	103	+ 3	9 3	122	+22
1901	26,897	108	+ 8	8.1	87	—13
1902	26,940	100	0	8.2	101	+ 1
1903	26,952	100	0	12.2	149	+49
1904	31,350	116	+16	8.7	71	—29
1905	27,205	87	—13	10.9	125	+25
1906	31,301	115	+15	10.0	92	— 8
1907	29,848	95	— 5	11.5	115	+15
1908	32,493	109	+ 9	9.2	80	—20
1909	31,060	96	— 4	14.3	155	+55
1910	32,467	105	+ 5	14.7	103	+ 3
1911	36,045	111	+11	9.7	66	—34
1912	34,283	95	— 5	12.0	124	+24
1913	37,089	108	+ 8	. .	. .	. .

change form to the results of the law of demand in the form of progressive ratios, so, in case of the empirical law of supply, a similar transition may be made. If, in the above equation, which expresses the relation between percentage changes, we put $y = Y - 100$, $x = X - 100$, we get

$$Y = .375X + 99.01,$$

which is the law of variation of acreage with price, expressed in terms of ratios.

We come now to the consideration of the law of supply of cotton when there is an agreement between the producers to restrict the acreage. We wish especially to ascertain whether it is possible so to determine the empirical law of supply, under conditions of restricted acreage, that we may estimate the amount of output which will afford the planters the maximum return for their expenditure of labor and capital. In order to narrow our inquiry and to illustrate a particular method of dealing with the problem, let us seek to determine in a concrete case what prices are necessary in order to call forth production of definite, indicated quantities of cotton. In Table IV is an arrangement of data which I have compiled from *Bulletin No. 659* of the United States Department of Agriculture, entitled "A Farm Management Study of Cotton Farmers of Ellis County, Texas." The Bulletin was published June 17, 1918, and its conclusions are "based on data obtained in a survey of the business of 120 farms in Ellis County, Texas, in 1914" (p. 1).

This county not only is one of the oldest agricultural sections of Texas but is also "one of the leading cotton sections of the state" (p. 14). Table IV, I should like to give in greater detail, but it is as full as the Tables from which it is derived[1] will admit. For the use we shall make of the data it would be highly desirable to have a much finer classification of the farms according to yield per acre, and perhaps it may be useful to suggest that future official investigations along similar lines will be greatly increased in value if the classifications are more detailed; are kept equidistant and constant in the range of the subclasses; and are retained in investigations covering different parts of the Cotton Belt so that many sample inquiries may be compounded into a representative index. This suggestion will not, I hope, be regarded as an ungracious criticism of an exceptionally able and useful report.

Returning, now, to Table IV, we see that in Group 1, which according to column VIII produces 18.6 per cent. of the total yield of the 115 farms, the average cost of producing cotton lint is 10.5 cents a pound (column IX). In a similar manner, the cost of production in each of the four yield-groups is given, and we see by a comparison of columns III and IX that there

---

[1] *Bulletin No. 659*, pp. 34, 35.

TABLE IV

DATA RELATING TO THE PRODUCTION OF COTTON IN ELLIS COUNTY, TEXAS

I Group	II Yield of Cotton Lint per Acre (Pounds)	III Average Yield of Cotton Lint per Acre (Pounds)	IV Number of Farms	V Average Crop Acres per Farm	VI Mean Yield per Farm (Pounds of Lint)	VII Total Yield of the Group (Pounds of Lint)	VIII Percentage of Product in Respective Groups	IX Average Cost of Production per Pound of Lint (Cents)
1	136 to 208	170	31	120	20400	632,400	18.6	10.5
2	209 to 240	227	33	133	30191	996,303	29.4	8.6
3	241 to 277	275	25	127	34925	873,125	25.8	8.2
4	278 to 402	345	26	99	34155	888,030	26.2	6.7
Total	. . . .	. . . .	115	. . . .	. . . .	3,389,858	100.0	

is a crude inverse relation between the yield per acre and the cost per pound of lint: *The greater the yield per acre the smaller the cost per pound of lint.* This crude relation gives us a clew to the path to follow in the search for the empirical law of supply. By an inspection of Table IV we find the following: If a full crop is to be produced the average cost of production of the marginal group, 10.5 cents per pound of lint, must be covered, and this cost is 93.7 per cent. of the average price,—11.2 cents a pound,—received for the five years previous to 1914 (Bulletin, p. 14); if the crop is to be reduced by agreement 18.6 per cent. (column VIII), the average cost of production in the new marginal group, 8.6 cents (column IX), must be covered, and this is 76.8 per cent. of the average price of 11.2 cents; if the crop is to be reduced 48.0 per cent., the average cost of production in the marginal group is 8.2 cents, which is 73.2 per cent. of 11.2 cents; finally, if the crop is to be reduced 73.8 per cent., the average cost of production in the marginal group is 6.7 cents, which is 59.8 per cent. of 11.2 cents.

Supposing that these percentages, instead of being derived from a single county in Texas, were the mean percentages of many samples from different parts of the Cotton Belt, we should be able to construct from them a general empirical law of supply which could immediately be brought into relation with the empirical law of demand. We shall illustrate how this might be done. The law of demand for cotton based upon the material of Table I is

$$Y = 173.6 + .377X - .01603X^2 + .0000572X^3.$$

If in this equation we put $X = 100$, we get $Y = 108.2$, which shows that there is a secular trend in prices tending to increase by 8.2 per cent. annually. Our empirical law of supply may be brought into relation with this law of demand only when we make a similar allowance for the secular change in the cost of production, and this has been provided for in the accompanying Table V. Columns III and V of this Table give the coördinates of four points in the empirical supply curve, and if we fit a parabola of the third order to these four points, we obtain as the equation to the empirical law of supply

$$Y = 1.0 + 3.842X - .06295X^2 + .0003457X^3.$$

The Economic Amount of Production

Our empirical laws of supply and demand, which are graphed in Figure 3, enable us to throw some light upon the method of answering the final question asked by the cotton planters, namely, what is the amount of production which, with the given laws of supply and demand, will yield the farmers the greatest income? If we express the above law of demand in the form $Y = F(X)$, and the above law of supply in the form $Y = \Phi(X)$, then the mathematical equivalent of the farmer's question is: For what value of X will $X[F(X) - \Phi(X)]$ be a maximum? By making the above substitutions for $F(X)$ and $\Phi(X)$, and solving the problem according to known mathematical rules, we find that the income of the farmers will have a maximum value when $X = 64.99$, that is to say, the farmers' greatest profit will be obtained when they restrict their output 35 per cent.

TABLE V

DATA FOR THE EMPIRICAL LAW OF SUPPLY OF COTTON

(Conditions of Restricted Competition)

I Group	II Percentage of Total Production Contributed by Groups	III Cumulative Percentage of Total Production	IV Cost of Production Expressed as Percentage of Selling Price	V Product of Column IV by 1.082
1	18.6	100.0	93.7	101.4
2	29.4	81.4	76.8	83.1
3	25.8	52.0	73.2	79.1
4	26.2	26.2	59.8	64.7

It will not be forgotten, I hope, that I have gone through

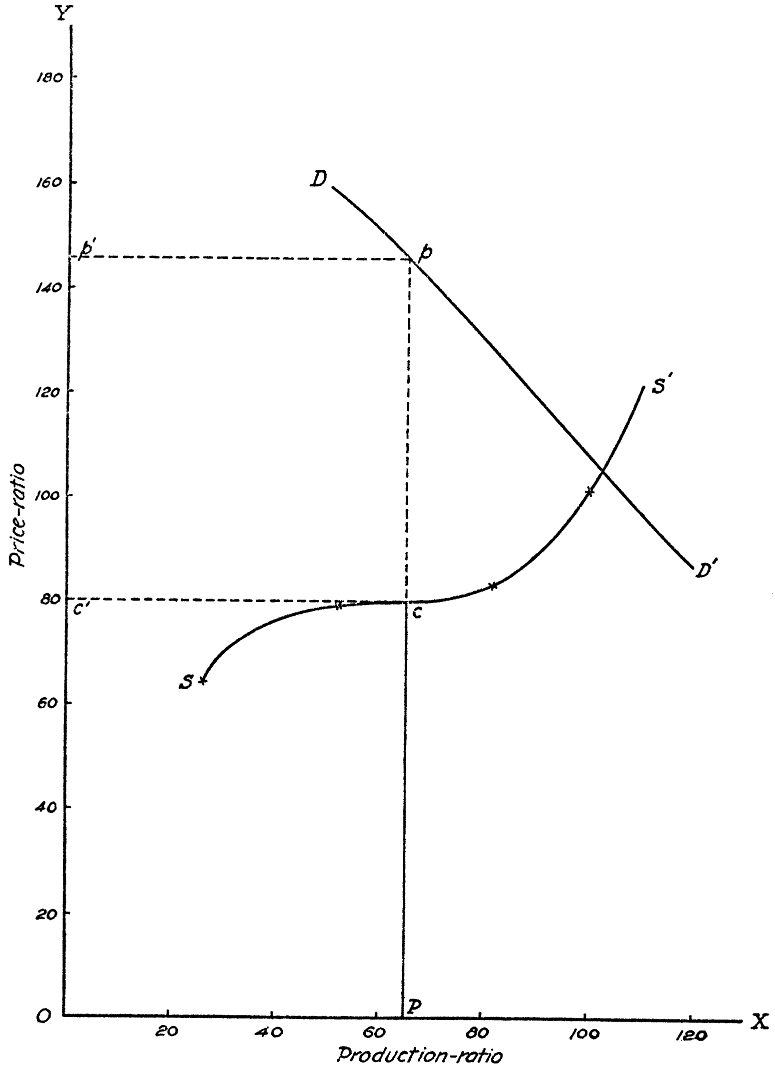

Figure 3. Laws of demand and supply in the case of cotton. (Conditions of restricted competition in production.)

Law of demand: $Y = 173.6 + .377X - .01603X^2 + .0000572X^3$

Law of supply: $Y = 1.0 + 3.842X - .06295X^2 + .0003457X^3$.

the details of this problem, primarily in order to illustrate a method of dealing with fundamental economic questions, and not to test the accuracy of the planters' propaganda aiming to bring about a reduction of their acreage by one-third. The remarkable coincidence between the above theoretical results and the estimates of the farmers might suggest, in the heat of controversy, that the numerical accuracy of the farmers' contention had been mathematically demonstrated. This, of course, is not true; first, because the empirical law of demand is based upon data ending in 1914; and secondly, because the empirical law of supply rests only upon a sample survey, in 1914, of one county of Texas. The propagandists, however, would doubtless declare, as to the empirical law of demand, that the experience of the twenty-five years before the World War is a most reasonable foundation upon which to base their plans in the period of reconstruction; and as to the law of supply, that faith in their recommendation would seem to be justified upon the assumption that the law of supply derived from a painstaking, official survey of conditions in our leading cotton-producing state is reasonably representative of the Cotton Belt.

HENRY L. MOORE.

COLUMBIA UNIVERSITY.